Final exam for Kwantumfysica 1 - 2004-2005

Thursday 24 August 2005, 14:00 - 17:00

READ THIS FIRST:

- Clearly write your name and study number on each answer sheet that you use.
- On the first answer sheet, write clearly the total number of answer sheets that you turn in.
- Note that this exam has 3 questions, it continues on the backside of the papers!
- Start each question (number 1, 2, 3) on a new answer sheet.
- The exam is open book. You are also allowed to use formula sheets etc.
- If it says "make a rough estimate", there is no need to make a detailed calculation, and making a simple estimate is good enough. If it says "calculate" or derive, you are supposed to present a full analytical calculation.
- If you get stuck on some part of a problem for a long time, it may be wise to skip it and try the next part of a problem first.

Useful formulas and constants:

Electron mass	m_e	$= 9.1 \cdot 10^{-31}$ kg
Electron charge	$-e$	$= -1.6 \cdot 10^{-19}$ C
Planck's constant	h	$= 6.626 \cdot 10^{-34}$ Js $= 4.136 \cdot 10^{-15}$ eVs
Planck's reduced constant	$\hbar$	$= 1.055 \cdot 10^{-34}$ Js $= 6.582 \cdot 10^{-16}$ eVs

Fourier relation between *x*-representation and *k*-representation of a state

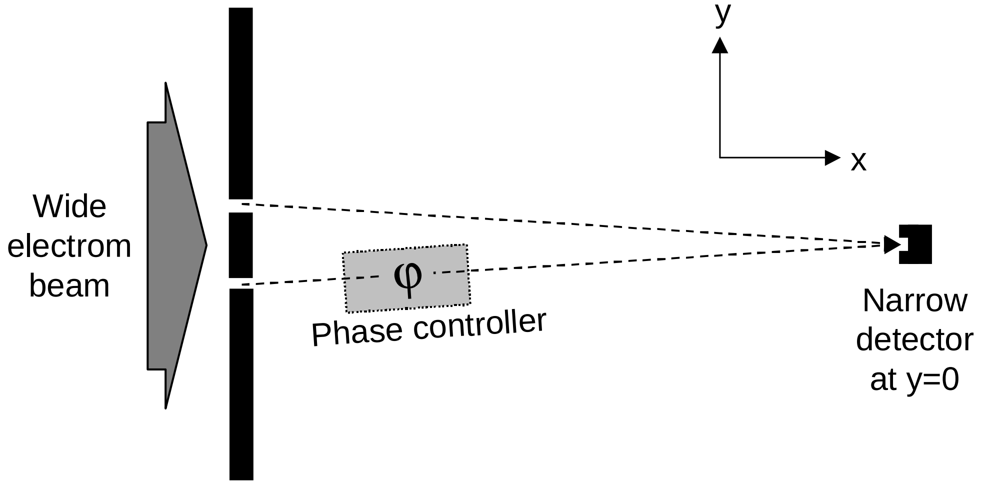
$$\Psi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \overline{\Psi}(k) e^{ikx}\, dk$$

$$\overline{\Psi}(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \Psi(x) e^{-ikx}\, dx$$

Z.O.Z.

Problem 1

Consider the double slit experiment in the figure. A very wide electron beam (diameter of the Gaussian profile is much larger than distance between the slits) is incident on a thin metal screen with two slits. The electrons in the beam are accelerated and arrive with a kinetic energy of 5000 eV at the screen. The screen is at $x = 0$ m. The slits are at $y = -100$ µm and $y = +100$ µm, and have a width of 1 µm. An electron detector is placed far from the screen at position $x = 1$ m, $y = 0$ m. In the path of one of the two slit-detector trajectories a phase controller is placed.



The phase (φ) controller is formed by two metal plates that lie parallel to the x-y plane, closely above and under the beam. The length in x-direction is L. By setting the same voltage V_φ on the two plates you can control the potential energy $-eV_\varphi$ that the electrons experience between the plates. Assume that this voltage will always be negative with respect to the grounded screen and experimental environment (so $-eV_\varphi$ is positive for $V_\varphi < 0$ V). The plates are very close together, and the electrons experience the entrance and exit of the phase controller as a sudden step-wise change in the potential.

a) Make a sketch of the potential that the electrons experience as a function of the distance x, between the screen and the detector, for a value $V_\varphi < 0$ V.

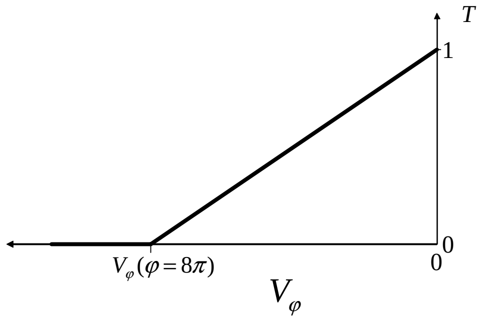
b) Calculate the de Broglie wavelength of the electrons before they enter the phase controller.

c) Derive an expression (use E_{k0} as notation for the electron kinetic energy before entering the phase controller) for the de Broglie wavelength of the electrons while they are inside the phase controller, with the voltage set at a value $V_\varphi < 0$ V, but with $-eV_\varphi < E_{k0}$.

d) Estimate the value of V_φ that is needed for setting an extra phase difference between the beams of $\varphi = 0$, $\varphi = \pi$, $\varphi = 2\pi$, and for $\varphi = 8\pi$. Assume $L = 1$ mm. Make a sketch of φ versus V_φ for the interval $V_\varphi(\varphi=8\pi) < V_\varphi < 0$ V.

e) Sketch the observed interference pattern in the detector signal as a function of $V_\varphi < 0$ V, with $-eV_\varphi < E_{kin}$, for the interval $V_\varphi(\varphi=8\pi) < V_\varphi < 0$ V. Assume here that there is no back reflection of electrons while entering or leaving the phase controller. The detector counts electrons, and the signal 1 nA for $\varphi = 0$.

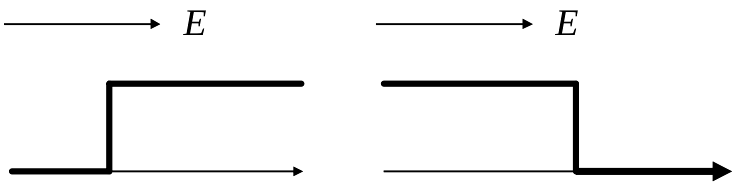
f) The experimentalists make some errors while building the phase controller. As a result, the electron transmission through the phase controller rapidly degrades when one decreases V_φ below 0 V. When doing a test they find the following relation between transmission coefficient T and V_φ.



Repeat question **e)**, but now show in the sketch what the influence is of the reduced transmission on the interference pattern. Explain how you get the answer by first calculating the amplitude of the detector signal for the phase controller set at $\varphi = 0$, $\varphi = 2\pi$, $\varphi = 4\pi$, $\varphi = 6\pi$, and $\varphi = 8\pi$.

Problem 2

The following figure schematically presents a particle incident on a simple step-up and step-down potential. The height of the step V and the total energy of the particle E is the same for the two cases. The lower value of the potentials is 0, and the higher value is V, and $E > V$.



a) For the particle that is incident the simple step-up potential (as in the left figure), the reflection coefficient R is

$$R = \left| \frac{1 - k_2 / k_1}{1 + k_2 / k_1} \right|^2,$$

where k_1 is the de Broglie wave number before the step, and k_2 that for after the step ($k = 2\pi/\lambda$). Derive and calculate R for the case that the particle is incident on the simple step-down potential (as in the right figure).

b) Compare the R for the step-up and step-down potential, and comment on why they are the same or different.

Problem 3

Note: where possible, use Dirac notation for solving this problem.

Consider a system with a time-independent Hamiltonian $\hat{H}$, that has only two energy eigenstates. These have two different energies E_1 and E_2 with $E_2 > E_1$.

a) Write down the time-independent Schrödinger equation for this system in Dirac notation.

b) This system has an electrical dipole moment that is described by the operator $\hat{D}$. This operator $\hat{D}$ does not commute with $\hat{H}$. Oscillations of $\langle \hat{D} \rangle$ in time at frequency f lead to the emission of electromagnetic radiation with frequency f. This occurs for example when the system makes a transition from level E_2 to level E_1. Show that this system can only emit radiation at frequency $f = (E_2 – E_1)/h$, by deriving the relation for the photon energy $hf = E_2 - E_1$.